WRC RESEARCH REPORT NO. 39

OPTIMAL PLANS FOR THE CAPACITY EXPANSION OF A MUNICIPAL WATER TREATMENT-DISTRIBUTION SYSTEM

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The work upon which this publication is based was supported by funds provided by the State of Illinois.

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ACKNOWLEDGEMENT

I am indebted to Professor Benjamin B. Ewing, Director of Water Resources Center, University of Illinois; his strong interest in the project has been my major source of encouragement from the time of writing the proposal to the time of completing the research.

I am particularly grateful to Mr. Robert H. Harmeson of the Illinois State Water Survey who has given me useful suggestions and technical information regarding water supply in general. Messrs. Robert S. Shierry and Charles H. Smith of Northern Illinois Water Corporation kindly supplied me with technical information regarding water distribution. And Mr. Richard A. Maltby of the Champaign County Regional Planning Office kindly helped me in obtaining the regional data necessary for the application of the model to the Champaign-Urbana area.

Mr. Goang-Tzer Liaw and Mrs. Lalit Kumar as my assistants have contributed so much to the successful completion of this project through their work in literature search and computation.

Finally, I acknowledge Mrs. Sharon Gocking for her patient and fine typing of the manuscript.

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Chapter 1

INTRODUCTION

In recent years, a growing number of studies has shown possibilities of applying mathematical optimization techniques to various water problems. Some of the studies merely formulate mathematical models using optimization techniques in the conceptual realm, while others not only present such models but demonstrate the models' operationality through examples. Among the optimization techniques, most commonly used are the linear programming and dynamic programming methods. Examples of the studies using the linear programming method are: the design of a deterministic river-basin system [Dorfman], the design of a stochastic reservoir system [Thomas and Watermeyer], water quality management [Revelle and others], the capacity expansion of sewage treatment facilities [Lynn], water treatment [Kerri], economics of water quality management [Johnson], and water pollution control in the Delaware Estuary [Thomann]. Examples of the subjects using the dynamic programming method include: the design of a multiple purpose reservoir [Hall, 1964], the determination of aqueduct capacity [Hall, 1963], water resource development [Hall and Buras], and multistage water resource systems [Meier and Beighter].

This study is an application of linear programming to the multi-stage capacity expansion of a municipal water treatment-distribution system, determining the sizes of treatment plants and distribution reservoirs and the points of time at which these facilities are installed. One of the salient points in capital investment is economics of scale available to large facilities. In this study, the scale effects are included in the capital and operating costs of facilities and given by concave functions of capacity. The original non-linear problem is transformed to a linear program by converting non-linear cost functions to linear functions in polygonal form. Then an optimal solution to the linear program is obtained approximating an optimal solution to the original non-linear formulation.

The plan thus developed may provide useful information for specifying facilities to be installed early in the planning period, preparing the acquisition of construction sites for facilities to be installed later in the period, or raising the funds required for such construction.

Chapter 2

GENERAL DISCUSSION

Before developing a mathematical model of the municipal water treatment-distribution system, a few salient aspects of the problem are discussed in brief.

Trend in Municipal Water Requirement

A long range plan for expanding the capacity of a water treatment-distribution system is preceded by the forecasting of future demand that takes into account past records of the type and pattern of community water use, physical and climatic conditions, expected housing, commercial and industrial developments, and trends of population increase. Significant factors determining water demand in a small residential community include number of residents, number of households, and density of dwelling units. According to a report submitted by the U. S. Senate Select Committee on Water Resources [p. 1], the population of the United States will grow from 108.9 million in 1959 to 204.4 million in 1980 and 321.8 million in 2000. In many small cities with rapidly growing residential districts this trend in population growth has been especially significant and is expected to continue for the foreseeable future.

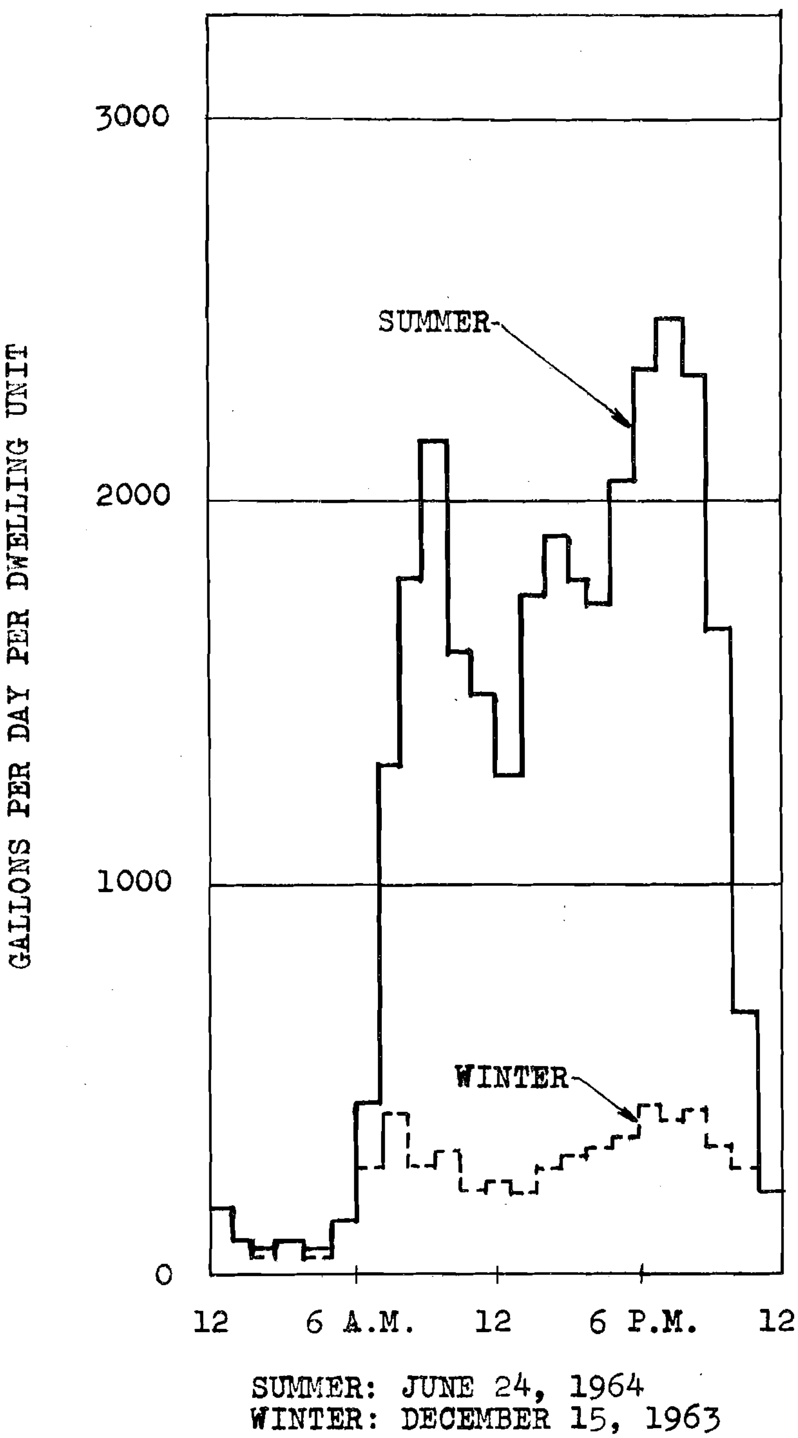
In addition to the possibility of a greater increase in population, other factors could have a considerable stimulating influence on the amount of water required in the future for municipal purposes. New household devices, new industrial uses, urban rearrangements, and continuing improvement in living standards could result in substantial increases per capita consumption. Also, there is a tendency of industries to purchase water from municipal systems rather than to maintain their own sources of supply. As wells and other sources available to them become polluted and destroyed, industries find it increasingly economical to seek water from public systems [U. S. Senate Select Committee, p. 11].

In summary, the Committee Report states that 147 gallons per capita per day average municipal use in 1960 may, under the circumstances and conditions described above, conceivably increase to about 185 gallons per capita per day in 1980, and to, perhaps, 225 gallons per capita per day in year 2000. Such requirements are strongly affected by regional and climatic conditions. In particular, where lawn sprinkling is widespread and prolonged, requirements may considerably exceed those figures [Clark, p. 33].

Patterns of Municipal Water Use

The rates of residential water use constantly fluctuate varying from hour to hour, day to day, and season to season. A study conducted by Linaweaver, et al. on 41 representative areas shows that maximum daily demands average 259 percent of the annual averages and peak hourly demands average 634 percent of the annual averages [Linaweaver, p. 13]. Most of difference between summer and winter use in residential areas is attributed to lawn irrigation. During the winter practically all of the water use is for domestic purposes inside the home. On a winter day, there are typically two peaks, one in the morning and one in the early evening. On a summer day a much higher peak occurs at about noon and often an even greater peak during the evening hours. As a specific example, the Linaweaver's study reports demand fluctuations in Creekside Acres, Oakland, California [Linaweaver, p. 9-10]. In this district, average summer use may exceed average winter use by a factor of three: on a hot, dry, sunny day, the sprinkling of lawns by a large number of consumers

FIGURE 1. COMPARISON OF HOURLY WATER USE ON TYPICAL SUMMER AND WINTER DAYS IN CREEKSIDE ACRES, OAKLAND, CALIFORNIA.



(Source: Linaweaver, et. al., A Study of Residential Water Use, p. 10.)

often impose a daily demand which is five times the average daily winter use. Hourly demand varies by an even wider margin. Figure 1 shows the demand patterns of a typical winter and summer day in Creekside areas reported by [Linaweaver, p. 10].

Water requirements of industrial users vary considerably with type of industry and characteristics of individual users and, therefore, accurate forecasting of industrial water requirements is extremely difficult. Although in the past many users developed their own supply systems, thus imposing no demand on the local municipal system, there is a trend that these users purchase water from municipal systems. Generally, commercial users of various types do not materially affect peak municipal demands. Maximum commercial needs are considerably less important than peak sprinkling demands in determining peak loads on a distribution system subject to heavy sprinkling loads [Clark, p. 37-38].

Estimation of Peak Demands

In determining the capacity of a water treatment-distribution system, determinant factors are the average annual demand, the maximum daily demand, and the peak hourly demand in a maximum day.

The existing FHA standards recommend designing for an average annual demand of 400 gpd per dwelling unit, a maximum daily demand of 800 gpd per dwelling unit, and a peak hourly demand of 2,000 gpd per dwelling unit, except 2,800 gpd per dwelling unit with extensive sprinkling. However, Linaweaver, et al. [p. 55] believe the above standards tend to lead to underdesign of systems in high-valued metered areas and overdesign in lower-valued metered areas and in apartment areas. Based on data obtained in 41 represntative areas in the United States, they suggest the following formula for determining the expected average demand [Linaweaver, p. 58-60].

(1) $\bar{Q} = (157 + 3.46V)a + 1.63 \times 10^4 a\bar{L}_s(\bar{E}_{pot} - \bar{P}_{eff})$

where $\bar{Q}$ = expected average demand for any period expressed as a rate in gallons per day

V = average market value in $1,000 per dwelling unit at 1964 price level

a = number of dwelling units

$\bar{L}_s$ = average irrigable area in acres per dwelling unit, specifically

$L_s = 0.803W^{-1.26}$

W = gross housing density in dwelling units per acre

$\bar{E}_{pot}$ = estimated average potential evapotranspiration for the period of demand in question in inches of water per day. In the absence of an exact value, $\bar{E}_{pot} = 0.28$ is recommended.

$\bar{P}_{eff}$ = amount of natural precipitation effective in satisfying evapotranspiration for the period and thereby reducing the requirements for lawn sprinkling in inches of water per day.

The expected maximum daily demand is obtained from (1) by setting $\bar{P}_{eff} = 0$:

(2) $\bar{Q}_{(mxdy)} = (157 + 3.46V)a + 1.63 \times 10^4 a\bar{L}_s\bar{E}_{pot}$

Variability in the expected maximum daily demand is a product of both the variability in the factors influencing demand and the diversity effect of the number of dwelling units. Taking these variations into consideration, Linaweaver and others recommend the following design daily demand, which has a 95 percent confidence interval or 2.5 percent chance of being exceeded--this would occur once in 40 years [Linaweaver, p. 60-64].

(3) $Q'_{(mxdy)} = \bar{Q}_{(mxdy)} + 2\sigma_{(mxdy)}a$

where

(4) $\sigma^2_{(mxdy)} = 1{,}090 + 1.66\ (10^4 L_s^{-2}) + 5.48\ (10^6)/a$

Finally, they propose the following $\bar{Q}_{(pkhr)}$ and $Q'_{(pkhr)}$ for determining the expected peak hourly demand and the design peak hourly demand with a 95 percent confidence interval, or a 2.5 percent chance of being exceeded:

(5) $\bar{Q}_{(pkhr)} = 334\ a + 2.02\bar{Q}_{(mxdy)}$

(6) $Q'_{(pkhr)} = \bar{Q}_{(pkhr)} + 2\sigma_{(pkhr)}\, a$

where

(7) $\sigma^2_{(pkhr)} = 4.04\{1{,}090 + 1.66\ (10^4 L_s^{-2})\} + 12.3\ (10^6)/a$

Water system supply and treatment facilities and most distribution facilities are designed from estimates of maximum daily and peak hourly demands. Although Linaweaver [p. 72] suggests as the design criterion the design peak hourly demand or the design maximum daily demand plus fire flow requirements, whichever is larger, these requirements seem too high. In practice, booster pumping accompanied by increased chloridation is used to increase the supply of water of a plant as much as 40% above its rated capacity on days of exceptionally large demands. Lowering the quality of water is unavoidable in such a case.

Requirements for Fire Fighting

In addition to demands created by residential, commercial and industrial uses, a municipal water system must satisfy requirements for fire-fighting.

The American Insurance Association (AIA), which has amalgamated the former National Board of Fire Underwriters (NBFU), recommends the following flow for the high-value district in an average municipality of 200,000 or less:

$$(8) \quad Q_t^* = 1.020 \sqrt{P} \quad (1-0.01 \sqrt{P}) \; 10^{-3}$$

where Q_t^* is demand in million gallons per minute and P is population in thousands. AIA further recommends the above fire flow to continue for the number of hours specified in Table 1. However, the specific value of a fire flow will be determined by the structural conditions and congestion of buildings in the district considered. Storage should be able to provide the required fire flow for the specified duration during a period of 5 days with consumption at the maximum daily rate. The maximum daily consumption is the maximum total amount used during any 24-hour period in the past 3 years. Where no figure for maximum daily consumption is available, its estimate should be at least 50 percent greater than the average daily consumption during the preceding year [NBFU, p. 14-32].

Table 1. Required Duration for Fire Flow*

Required Fire Flow Q_f gpm	Required Duration H_f hours
Less than 1,250	4
1,250 and greater, but less than 1,500	5
1,500 " " " " " 1,750	6
1,750 " " " " " 2,000	7
2,000 " " " " " 2,250	8
2,250 " " " " " 2,500	9
2,500 and greater	10

*From NBFU Grade Schedule, p. 20.

Water Distribution System

The water treatment-distribution system to be developed later encompasses specifically the water-treatment plant, including the pumping facilities, and the distribution reservoir. This system at the outset has one treatment plant and one distribution reservoir and will be added by more treatment plants and reservoirs as demand for water increases with time. These facilities are interlocked to one another and assumed to function as an integral unit.

The system must be designed to satisfy those maximum requirements discussed in the previous sections. Because of seasonal or hourly variations in water use pattern, considerable capacity is idle much of the time. The treatment plant and its associated facilities could have a sufficiently large pumping rate to satisfy directly a maximum demand rate at any point of time during a day, adjusting the pumping rate to changing demand. Such an operation requires the capacity of a plant which is not fully utilized most of the time. Further, it is usually economical to operate the pumping station at a constant rate, supplying a constant flow of water to the system throughout the operating hours. Excess supply of water during slack periods is stored in distribution reservoirs, either surface reservoirs or elevated tanks, and used to compensate for the insufficient flow of water from the treatment plant during peak periods or at times of extraordinary demand such as fire fighting.

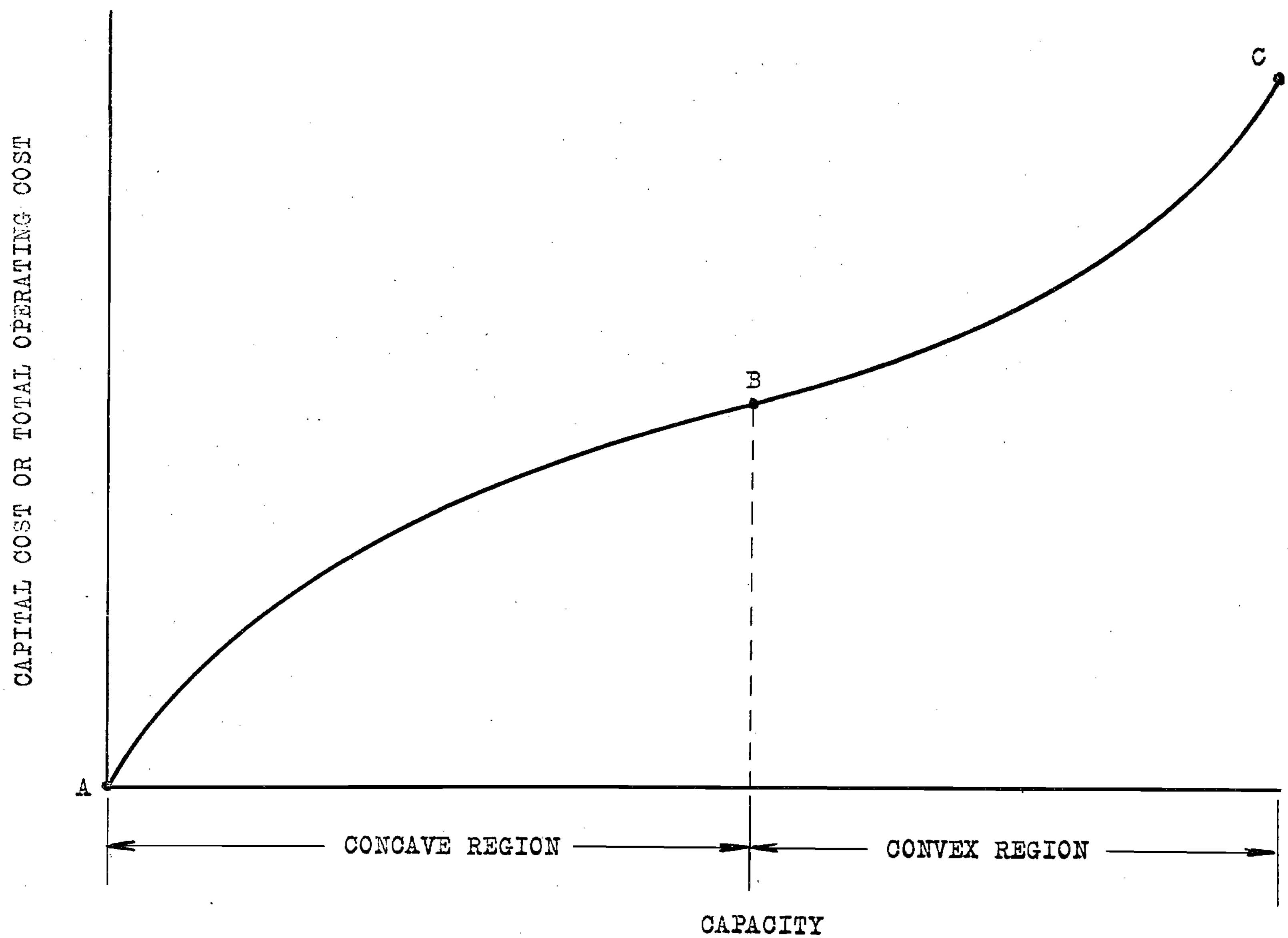
Under increasing demand, the installation of a treatment plant and associated facilities designed to meet immediate needs will sooner or later become inadequate. A tested and economically feasible plan in this situation is to build a sufficiently large plant that satisfy the demand for some years to come. Similarly, with demand increasing continuously, the system eventually needs an additional reservoir in order to make a better use of the installed plants. Thus the capacity of the water treatment-distribution system could be expanded by the mixed installation of treatment plants and reservoirs.

Regional conditions, the quality of available water, or total municipal demand determines whether water is obtained from underground sources or surface sources such as reservoirs, rivers, or lakes. The total investment and treatment costs of a treatment plant depend on the type of water source being used. As for distribution reservoirs, the most economic type is the ground-level reservoir or standpipe on adjacent hills. This type is infeasible where hills or elevated areas are absent. In flat areas, distribution storage takes the form of either elevated tanks or ground-level reservoirs with booster pumping stations. Noneconomic advantages and disadvantages of these reservoirs are discussed in some of the existing literature--see, for example, Keech.

Water Treatment Costs

One of the most important aspects in any capital investment is economies of scale related to the size or capacity of the capital facility. Normally, the scale effects are reflected on the capital cost of the facility and/or its operating cost. The most common relationship between capacity and such a cost is given by a concave-convex function showing a decreasing marginal cost with an increase in capacity up to a certain point, beyond which the marginal cost increases with an increase in capacity as is illustrated by the curve in Figure 2. Normally, capital investment analysts are interested only in the region of the decreasing marginal cost, i.e., the region under curve AB in Figure 2.

FIGURE 2. TYPICAL RELATIONSHIP BETWEEN CAPACITY AND CAPITAL COST OR TOTAL OPERATING COST FOR CAPACITY OPERATION



The total cost of water treatment, like a regular capital investment project, is composed of the capital cost of a plant and the costs of operation and maintenance. These costs are significantly influenced by both the capacity of the treatment plant and sources from which water is obtained.

Literature on the costs of water treatment is very limited. The following discussion is based mainly on information available from publications by Koenig and the Illinois State Water Survey.

The total treatment cost of surface water reported by Koenig is based on data from 30 plants. The capital cost of a plant covers the low lift pumping station, the treatment plant itself, and the high lift pumping station, but it does not include conveyance lines for raw water or finished water, nor booster stations on finished water lines or distribution lines [Koenig, p. 295]. The Illinois State Water Survey (ISWS) adjusted data from 42 plants (including Koenig's 30 plants and other data which appeared in the Journal of the American Water Works Association) to 1964 prices and to location differences by using the Handy-Whitman Utilities Indexes for small treatment plants, 0 to 1 million gallons per day (mgd), and large treatment plants, greater than 1 mgd. Using the adjusted data, ISWS then obtained the following regression relation between capacity and capital cost:

(9) $E_{ps} = 267.9K^{0.65}$ in \$1000

where E_{ps} is the capital cost in \$1,000 and K is the capacity in mgd.

Koenig's study [p. 324] indicates that the capital costs, amortized over 30 years at 4% interest rate, contributes the greatest portion of the total treatment costs, being 40-55% depending on the degree of plant-capacity utilization. The next major item is manpower, contributing 22% in typical plants. The third item in the list is energy with 10-13% contributions. These three items contribute almost 7/8 of the total cost. Other items included in the cost are chemicals with 6% contribution, heating, maintenance, and repair, each with 2% constribution. The total treatment cost of "typical plants" tabulated by Koenig is reproduced in Table 2, in which the heating cost represents about the maximum to be experienced in the United States.

Table 2. Elements of Average Water Treatment Cost in "Typical Plants" (1964 Price Level)

Design Capacity K mgd	0.5		8.0	
Utilization Rates	0.5	1.0	0.5	1.0
Item	$/mg	$/mg	$/mg	$/mg
Manpower	67.0	61.0	27.0	16.0
Maintenance, Repair and Replacement	7.0	5.0	2.9	1.7
Miscellaneous	2.2	1.1	2.2	1.1
Heating (140 days)	9.2	4.6	2.2	1.1
Energy	33.0	33.0	17.0	17.0
Chemicals	18.5	18.5	7.2	7.2
Average Operating Cost f(u)	136.9	123.2	58.5	44.1

Source: Koenig [21, p. 324]

In computing a unit cost of water treatment, Koenig allocated the annual amortization of capital cost, along with other costs, to each gallon of water treated at a given rate of plant utilization. The result is the average cost of water treatment at that utilization rate. Because the average cost changes as the utilization rate shifts, the cost obtained by the above method is not useful for planning the long-range capacity expansion where the capacity of the system in each period is an undetermined decision variable. Specific information needed for the investment analyst in this case is the operating cost given as a function of capacity and utilization rate. Such information, however, is not available in the existing literature.

Since a concrete cost function is essential for computing numerical examples with the formulation subsequently developed, this investigator has derived a tentative cost function for surface water treatment from Koenig's cost data listed in Table 2. The details of this derivation is discussed in Appendix I. The derived function representing the total annual operating cost is given by the following $H_p(K, u)$ for a plant with capacity K(mgd) operated at utilization rate $u(0 \leq u \leq 1)$:

$$(10) \quad H_p(K, u) = 5.06\ K^{1.02}\ (1-u) + 34.79\ K^{0.63}\ u \qquad \text{in \$1000/Yr.}$$

This total operating cost is composed of the fixed and variable parts. The fixed part is incurred regardless of the rate of utilization and can be determined by replacing u in (10) with 0. The difference between $H_p(K, u)$ and the fixed cost thus determined represents the total variable cost. Then the annual fixed and variable costs of the surface-water treatment plant with capacity K (mgd) operated at u are given by the following $F_p(k)$ and $G_p(k, u)$:

$$(11\text{-}1) \quad F_p(K) = 5.06\ K^{1.02} \qquad \text{in \$1000/Yr.}$$

$$(11\text{-}2) \quad G_p(K, u) = u(34.79\ K^{0.63} - 5.06K^{1.02}) \qquad \text{in \$1000/Yr.}$$

The Illinois State Water Survey reported the cost of ground water treatment based on data from 58 plants located in Illinois [16, Tech. Letter 11]. These data were adjusted to 1964 price levels, and additions were made for engineering, legal, administrative, and overhead costs plus interest during construction. Then ISWS obtained the following regression of investment costs on plant capacity with 33.5 as the percentage measure of dispersion by the standard error of estimate:

(12) $E_{pg} = 115K^{0.63}$ in \$1000

where E_{pg} is the plant cost and K is capacity in mgd.

Becuase of its low degree of impurity, ground water generally requires much less work, and therefore cost, in treatment than surface water. Unfortunately we have found no published studies that give operating costs of the ground water treatment plant as detailed as those reported by Koenig on the surface water treatment plant.

ISWS reported the total treatment costs of the two types of plants in which the capital cost of the plant takes up an identical proportion of the total cost. Assuming this situation generally exists, we introduce a coefficient, δ, to covert various operating costs of the surface water plant to those of the ground water plant. Then the annual total, fixed and variable costs of operating the ground water plant are given by the following $H_p'(K)$, $F_p'(K)$, and $G_p'(K)$ based on those costs of the surface water plant in (10)-(11):

(13) $H_p'(K) = \delta\{5.06K^{1.02}(1-u) + 34.79K^{0.63}\ u\}$

(14-1) $F_p'(K) = \delta(5.06K^{1.02})$

(14-2) $G_p'(K,u) = \delta u(34.79K^{0.63}-5.06K^{1.02})$ in \$1000/Yr.

The conversion coefficient δ may be estimated from the capital cost functions of the surface water and ground water plants previously obtained in (9) and (12) as follows:

$$(15) \qquad \delta = \frac{115K^{0.63}}{267K^{0.65}} \approx .43$$

Cost of Distribution Storage

Through the distribution main, treated water pumped out of the plant reaches the community where it is consumed for immediate use during the peak demand period. During the slack period, however, the excess capacity of the treatment plant over the immediate demand is used to supply water for distribution storage; surplus water thus delivered is pumped into distribution reservoirs. Two types of reservoirs are commonly used for this purpose: the elevated tank and the ground-level reservoir. Once stored in distribution reservoir, water can be used to equalize the discrepancy between the demand and the direct supply from treatment plants during the periods of peak demand or emergency needs such as fire fighting. Water reaches the point of use by gravitation, if it is stored in an "elevated tank", or by booster pumping, if it is stored in a ground-level reservoir.

Economies of scale should affect the investment decision on selecting a reservoir, whether of the elevated type or of the ground-level type. Literature on the capital and operating costs of distribution storage is extremely scarce. In absence of other data on hand, the linear cost function reported by Keech[1] [20, p. 105] is converted to a concave function in order to give a scale

[1] Keech's data are used strictly for an illustrative purpose, because they are reported without detailed information substantiating their reliability.

effect in the capital cost. This is done not for improving the given cost function, but for creating a plausible function to be used in the numerical example discussed later. First, two capital costs are obtained at both ends of each of the straight lines given by Keech for the elevated tank and the ground-level reservoir. These costs are converted from the 1961 price level used by him to the 1964 price level by Handy-Whitman Index to be compatible with the costs of water treatment previously presented. Using the converted prices, the following costs functions are obtained:

$$(16)\quad E_s(K) = \begin{cases} 193.71K^{.786} & \text{(the elevated tank)} \\ 128.19K^{.751} & \text{(the ground-level steel reservoir with a pumping station)} \end{cases}$$

in \$1,000

where K is the capacity of the storage unit in mg.

The total operating cost of distribution storage covers pumping, painting and maintenance. The cost of pumping is a function of the volume of water pumped in or out of storage, while the costs of painting and maintenance are relatively fixed expenditures. The Illinois State Water Survey[1] estimated the cost of pumping water to be

$$(17)\quad C_{pump} = 31.4\ c/E_o \text{ per } 1{,}000 \text{ gallons/100 ft.}$$

where c is \$ per Kw-hr and E_o is the wire-water efficiency in percent.

The annual costs of painting and maintenance may be considered as fixed costs. Using the method employed in obtaining the capital costs in (16), those fixed costs given by Keech in straight lines are converted to the following concave functions showing economies of scale:

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[1]The Illinois State Water Survey, Technical Letter 9, Cost of Pumping Water, July 1968.

$$(18)\quad F_s(K) = \begin{cases} 920.18K^{.634} & \text{(the elevated tank)} \\ 757.2K^{.711} & \text{(the ground-level storage with a pumping station)} \end{cases}$$

in \$/mg.

Chapter 3

FORMULATION OF PROBLEM

Preliminary Conditions

The determination of a planning period for an investment project without a specific life is one of the annoying problems that face the capital investment analyst.

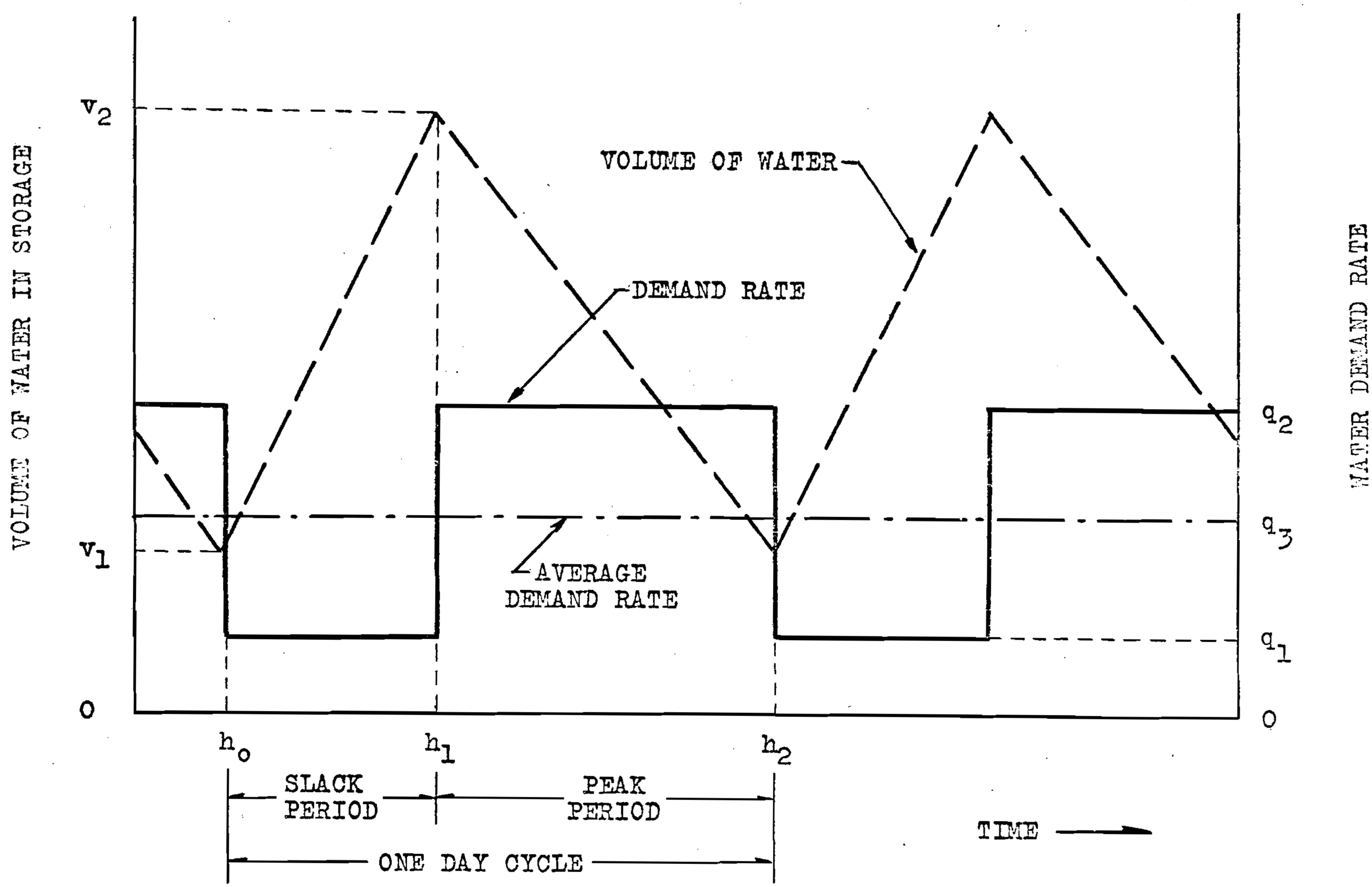
An arbitrary finite period might greatly influence the result of the analysis, because it implicitly assumes that the project at the end of its life will be succeeded by investment earning interest at the rate used for discounting. To avoid this difficulty, some authors suggested to use an infinite time horizon and to install an infinite series of either identical facilities or facilities whose capacities change in systematic manner, assuming that all conditions are either stationary or otherwise they change with time in well behaved manner.[1] Although this study adopts the concept of a permanent chain of identical facilities, it uses a finite period for initiating such chains.

Specifically, the study covers a finite period of T years for planning the capacity expansion of an existing water treatment-distribution system to satisfy increasing demands for water. This period may be considered either as the maximum length of time for which demand can be forecast with reasonable accuracy or as the period of increasing demand beyond which demand will become stationary. The facilities installed before the end of the period will be replaced at the ends of their optimum lives by permanent chains of identical facilities.

The peak period in each day can be separated clearly from the rest of the day because of a steep ascent in demand in the morning and of a steep

[1] One of the first authors to suggest a permanent chain of facilities is G.A.D. Preinreich; "Economic Life of Industrial Equipment," Econometrica, 8, 12 (July 1940), p. 12-44.

FIGURE 3. WATER DEMAND RATES IN SLACK AND PEAK PERIODS AND VARIABLE VOLUME OF WATER IN DISTRIBUTION STORAGE.



descent in the evening, as is observed in Figure 1. For the purpose of analysis, demands in each day are divided between the peak and slack periods of constant requirements. Further, following a common practice, the system is to operate at a constant rate throughout the day. In Figure 3, q_1 and q_2 show the constant demand rates in the slack and peak periods of a typical day; the constant rate of plant operation is given by q_3; and points h_1 and h_2 show the start and end of the peak period, respecitvely. The above difference between the demand and supply rates in the course of a day creates a variable amount of water stored in distribution storage, which is shown by a broken line. It is convenient to regard the end of the peak period, or point h_0 in Figure 3, as the start of the daily cycle. Because of the excess pumping rate over the demand rate during the slack period, the volume of water in storage steadily increases with time from v_1 at time h_o; it reaches the maximum volume v_2 at time h_1, the beginning of the peak period; and then it diminishes continuously during the peak period and goes down to the minimum volume v_1 again at time h_2, the end of the peak period, thus completing one cycle. The minimum volume v_1 should at least satisfy the fire fighting requirements.

The first task is to determine the expected average annual demand $\overline{Q}_t$ mgd, the expected maximum daily demand $\overline{Q}_{(mxdy)\,t}$ mgd, the fire flow Q^*_t mg by using (1), (2), and (8), or some other methods for the t^{th} year ($t = 1, \cdots, T$). The system will be designed to satisfy the AIA recommendations [NBFU, p. 16] specifying that the daily delivery capability should satisfy Q^*_t plus the specified maximum demand Q_t being equal to $\overline{Q}_{(mxdy)\,t}$ or 1.5 $\overline{Q}^*_t$

whichever is larger:

(19) $Q_t = \max\ (\overline{Q}_{(mxdy)t}\ ,\ 1.5\overline{Q})$ in mgd

Demand beyond the T-year period is assumed to stay equal to Q_T:

$Q_t = Q_T$ $t = T+1,\ T+2,\ \ldots$

Original Formulation

The existing capacity of water treatment is X_o mgd and that of distribution storage is Y_o mg. Decision variables in this analysis are X_t and Y_t denoting the capacities of a plant and reservoir installed at the beginning of the t^{th} year. Since no disinvestment of installed facilities is considered, the decision variables should take non-negative values.

(20) $X_t,\ Y_t \geq 0$ $t = 1,\ \ldots,\ T$

It is expected that the sum of the rated capacities of the plants in the system should satisfy at least the specified maximum demand Q_t with the coefficient of booster pumping ϕ:

(21) $\Sigma_{j=0}^{t}\ \phi X_j \geq Q_t$ $t = 1,\ \ldots,\ T$

Similarly, the total storage capacity of the reservoirs in the system should not be less than the volume of water required for both the fire fighting and the peak-period equilization on the maximum demand day. This is given by the following constraint:

(22) $\Sigma_{j=0}^{t}\ Y_j \geq Q_t^* + \beta Q_t - \phi\alpha\ \Sigma_{j=0}^{t}\ X_j$ $t = 1,\ \ldots,\ T$

where α and β are fractional ratios representing the length of time in the peak demand period over the day and the requirement in this period over the total requirement of the maximum day, Q_t.

The parameter ϕ is introduced to close the gap between theory and practice. Design formulas for plant capacity normally suggested in textbooks are based on the maximum-day demand and fire-fighting requirements. However, it seems in actual practice that the capacity is determined on a more conservative basis such as the average demand in a peak season, or that the plant output rate is boosted above the rated capacity, by as much as 40% in some cases, when the requirements are exceptionally high. The capacity determined by various design formulas is aimed at assuring a high quality of the treated water at all times. Such a quality, with a possible exception of chloridation, may have to be sacrificed at times of exceptionally big demand if the plant has to be boosted much higher than its rated capacity over a long duration.

The equality of constraint (22) holds at the time when the total requirement requires the operation of all the plant at full capacity. For demand less than that amount, the plants need not to operate at capacity levels. In this situation, if the plants are to operate at constant rates throughout the day, the volume of water required for equilization in the peak period is $(\gamma-\alpha)\overline{Q}_t$ mg on a typical day where γ is the fraction of the day's demand required during the peak period. If a major fire breaks out on such a day and the fire-fighting requirement Q_t^* must be satisfied over the period of 10 hours as specified by AIA, it is necessary to increase the output rates of the plants to their maximum levels. If any additional supply is necessary to meet Q_t^*, it must come from the reservoirs. Therefore, the following constraint on the storage capacity must be satisfied:

$$(23) \qquad \Sigma_{j=0}^{t} Y_j \geq (\gamma-\alpha)\overline{Q}_t + Q_t^* - \frac{10}{24}(\phi\Sigma_{j=0}^{t} X_j - \overline{Q}_t) \qquad t=1, \ldots, T$$

This constraint tends to require a larger reservoir capacity than that required by (22) in those years when the plants in the system are not fully used.

Finally, at a constant rate of operation, the capacity of distribution storage must satisfy at least the volume required for equalization during the peak period on an average day. This is written to

$$(24) \qquad \Sigma_{j=0}^{t} Y_j \geq (\gamma-\alpha)\overline{Q}_t \qquad t=1, \ldots, T$$

Cost Functions

The objective of this plan is to minimize the sum of the present value of the capital and operating costs of treatment plants and distribution reservoirs, discounted at a given interest rate. Since demands must be satisfied, this cost minimization is identical with the maximization of the present value of a stream of future net revenues. The total cost is given by

$$C_{tot} = \begin{bmatrix} C_{pc} \\ \text{Capital Costs} \\ \text{of plants} \end{bmatrix} + \begin{bmatrix} C_{sc} \\ \text{Capital Costs} \\ \text{of reservoirs} \end{bmatrix} + \begin{bmatrix} C_{po} \\ \text{Operating Costs} \\ \text{of plants} \end{bmatrix} + \begin{bmatrix} C_{so} \\ \text{Operating Costs} \\ \text{of reservoirs} \end{bmatrix}$$

By assumption, facilities invested during the finite planning period initiate infinite chains of facilities extending beyond the period. The facilities in each chain being identical, their capital costs amortized over their lives form a permanent series of an identical cost. The capital costs of a plant with capacity X mgd and a reservoir with capacity Y mg are given by $E_p(X)$ and $E_s(Y)$ in \$1000. Then the present values of the capital costs of all plants and reservoirs in the permanent chains initiated during the planning period are given by the following C_{pc} and C_{sc}, respectively:

$$(25) \qquad C_{pc} = \Sigma_{t=1}^{T} \frac{a\, E_p(X_t)}{(1+R)^{t-1}(1-\frac{1}{1+R})} \qquad \text{in \$1000}$$

$$(26) \qquad C_{sc} = \Sigma_{t=1}^{T} \frac{b\, E_s(Y_t)}{(1+R)^{t-1}(1-\frac{1}{1+R})} \qquad \text{in \$1000}$$

where R is the discount rate, a and b are the amortization factors for plants and reservoirs, the first term in the denominator is for discounting the costs to the present, and the second term is to convert the numerator to the discounted value of a permanent series of annual costs identical with this numerator. In Eqs. (25) and (26), the capital costs of facilities succeeding the existing ones are not included since they are not part of the decision made in this study.

The annual operating cost of a treatment plant is composed of the fixed cost and total variable cost and charged at the middle of that year. The annual fixed cost of a plant with capacity X is denoted by $F_p(X)$, and the annual variable cost at capacity operation is denoted by $G_p(X)$. To simplify the model, the existing system is assumed to have one treatment plant with capacity X_o. Demand for water on an average day is first allocated to new plants and then any remaining requirement is allocated to the existing plant. Therefore, the new plants are usually kept busy, while the existing plant may have slack capacity. Then the present value of the total operating costs of the permanent chains of plants in the system is given by the following C_{po}:

$$(27) \quad C_{po} = \frac{F_p(X_o) + G_p(X_o)}{(1+R)^{1/2}(1-\frac{1}{1+R})} + \Sigma_{t=1}^{T} \left\{ \frac{F_p(X_t) + G_p(X_t)}{(1+R)^{t-1/2}(1-\frac{1}{1+R})} \right\}$$

$$- \Sigma_{t=1}^{T} \left\{ \frac{\Sigma_{i=0}^{t} X_i - \overline{Q}_t}{(1+R)^{t-1/2}} \right\} \frac{G_p(X_o)}{X_o} - \left\{ \frac{\Sigma_{i=0}^{T} X_i - \overline{Q}_T}{(1+R)^{T+1/2}(1-\frac{1}{1+R})} \right\} \frac{G_p(X_o)}{X_o}$$

where the first term is the present value of the operating costs of the existing plant and its successors, the second term represents the present value of the total operating costs for the chains initiated by facilities installed during the period, the third term gives downward adjustments in the variable operating costs due to slack capacity computed at $G_p(X_o)/X_o$ representing the unit variable operating cost for the existing plant, and the fourth term is downward adjustments in the variable operating costs for years beyond the finite period.

It is assumed that water required for equalization during the peak period on an average day is stored in the reservoirs in proportion to their storage capacities, and that the system has only one existing reservoir with capacity Y_o at the outset. The fixed and variable operating costs of a reservoir with capacity Y is denoted by $F_s(Y)$ and $G_s(Y)$. The present value of the total operating costs of the permanent chains of reservoirs are given by the following C_{so}:

$$(28)\qquad C_{so} = \frac{F_s(Y_o)}{(1+R)^{1/2}(1-\frac{1}{1+R})} + \Sigma_{t=1}^{T}\left\{\frac{F_s(Y_t)}{(1+R)^{t-1/2}(1-\frac{1}{1+R})} + \frac{(\gamma-\alpha)\overline{Q}_t}{(1+R)^{t-1/2}}\ \frac{\Sigma_{i=0}^{t}\ G_s(Y_i)}{\Sigma_{i=0}^{t}Y_i}\right\} + \frac{(\gamma-\alpha)\overline{Q}_t}{(1+R)^{T+1/2}(1-\frac{1}{1+R})}\ \frac{\Sigma_{i=0}^{T}\ G_s(Y_i)}{\Sigma_{i=0}^{T}Y_i} \qquad \text{in \$1000}$$

where α and β are coefficients representing the time and demand belonging to the peak period as fractions of the time and demand on an average day of the year. Thus $(\gamma-\alpha)\overline{Q}_t$ gives the volume of water supplied from reservoirs during the peak period when the system treats water at a constant rate throughout the day. In Eq. (28), the first term is the fixed operating costs of the existing reservoir and its successors, the term on the lefthand side in the braces is the fixed operating costs of the reservoirs installed during the planning period and their successors, the term on the righthand side in the braces is the variable operating costs incurred during the period, and the third term is the variable operating costs for years beyond the planning period.

The total cost of the decision on an expansion plan is the sum of the capital costs C_{pc} and C_{sc} and the total operating costs C_{po} and C_{so}, given by (25) - (28). The variable operating cost of a reservoir is mostly for pumping and very close to a linear function of pumpage regarded almost independent of the capacity of the reservoir. Thus the total cost is given by

$$(29) \quad C_{tot} = \Sigma_{t=1}^{T} \left\{ \frac{aE_p(X_t) + bE_s(Y_t)}{R(1+R)^{t-2}} + \frac{F_p(X_t) + G_p(X_t) + F_s(Y_t)}{R(1+R)^{t-3/2}} + \frac{c\,\Sigma_{i=1}^{t} X_i}{(1+R)^{t-1/2}} \right\} - \frac{c\,\Sigma_{i=1}^{T} X_i}{R(1+R)^{T-1/2}} + D \qquad \text{in \$1000}$$

where

$$c = G_p(X_o)/X_o \qquad \text{in \$1000}$$

and D is the costs independent of the decision, given by the following:

$$(30) \quad D = \frac{(1+R)^{1/2}}{R}\left\{F_p(X_o) + G_p(X_o) + F_s(Y_o)\right\} - \Sigma_{t=1}^{T}\frac{c(X_o-\overline{Q}_t)}{(1+R)^{t-1/2}}$$

$$- \frac{c(X_o - \overline{Q}_T)}{R(1+R)^{T-1/2}} + d(\gamma-\alpha)\left\{\Sigma_{t=1}^{T}\frac{\overline{Q}_t}{(1+R)^{t-1/2}} + \frac{\overline{Q}_T}{R(1+R)^{T-1/2}}\right\} \quad \text{in \$1000}$$

where

$$d \approx \frac{\Sigma_{i=0}^{t} \; G_s(Y_i)}{\Sigma_{i=0}^{t} \; Y_i} \qquad t=1, \ldots, T$$

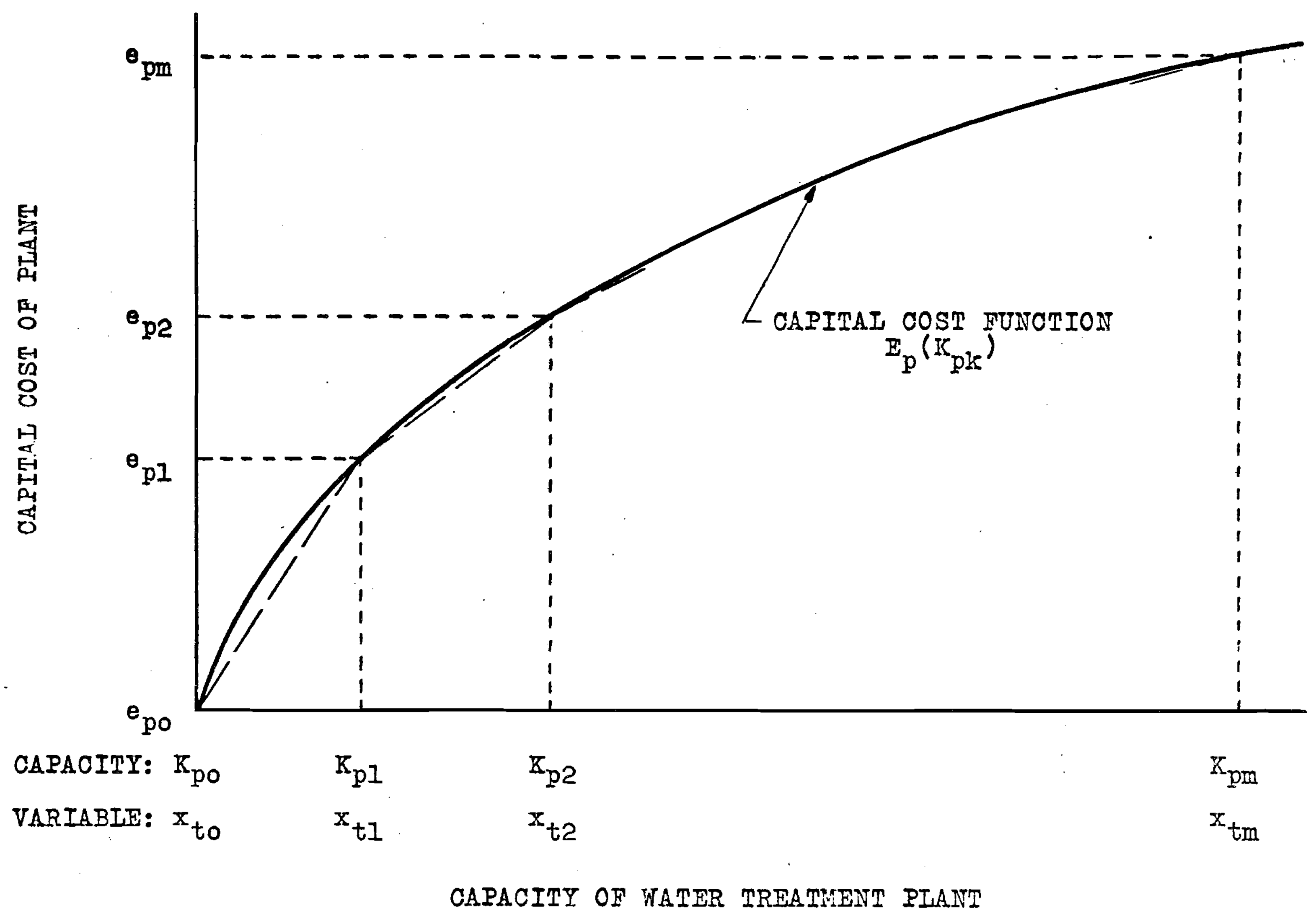
This d represents the average, variable operating cost per mg of all the reservoirs in the system. Assuming the main part of the variable cost is due to pumpage, in the subsequent numerical example d is replaced by the cost of pumping given by (17).

In the total cost C_{tot} in (29), all individual costs are non-linear functions of capacity; specifically they are given concave exponential forms to properly reflect economies of scale. The non-linear forms of these functions make them unacceptable for linear programming and will be converted to acceptable forms.

Linear Program

A nonlinear problem can be translated to a linear program, if all nonlinear functions can be represented by separate functions involving a single variable. An optimal solution to the linear program approximates an optimal solution to the original nonlinear problem. In particular,

FIGURE 4. POLYGONAL APPROXIMATION OF NON-LINEAR COST FUNCTION OF WATER TREATMENT PLANT.



if all nonlinear functions satisfy either concave or convex properties, the solution for the linear program represents the global optimum; otherwise, the solution normally represents a local optimum. For the transformation of a nonlinear function to a linear function, two methods are frequently mentioned in literature[1]: One is the cummulative gradient method and the other is the method using the interpolation of specific values of the function. The latter method is used to formulate the present problem.

First, the capital-cost function for the plant is transformed to a functional of linear functions with a single variable. For this, a range of capacity for possible interest is divided into m intervals giving a desired level of approximation. Specific values of capacity dividing the range are K_{pk} (k=0, 1, ..., m), as is shown in Figure 4. Variable X_t is now transformed to a set of variables x_{tk} associated with K_{pk} in the following way:

(31) $X_t = \Sigma_{k=0}^{m} K_{pk} x_{tk}$ $\qquad t = 1, \ldots, T$

where

(32) $\Sigma_{k=0}^{m} x_{tk} = 1$

(33) $x_{tk} \geq 0$

The capital cost, $E_p(X_t)$, and the fixed operating cost, $F_p(X_t)$, are now replaced by the following approximations using variables x_{tk}:

[1]See for example, Charnes and Cooper (p. 348-408), Dantzig (p. 482-490), Hadley (10, p. 104-147), Miller (p. 89-100) in Graves and Wolfe, or Dorfman, et. al. (p. 497-508).

(34) $\hat{E}_p(X_t) = \Sigma_{k=0}^{m} e_{pk} x_{tk}$ $\qquad t = 1, \ldots, T$

(35) $\hat{F}_p(X_t) = \Sigma_{k=0}^{m} f_{pk} x_{tk}$

(36) $\hat{G}_p(X_t) = \Sigma_{k=0}^{m} g_{pk} x_{tk}$

where

$e_{pk} = E_p(K_{pk})$ $\qquad k = 0, 1, \ldots, m$

$f_{pk} = F_p(K_{pk})$

$g_{pk} = G_p(K_{pr})$

For illustration, Figure 4 shows the relationship between K_{pk} and e_{pk}.

It is hoped that an optimal value of X_t is given by a convex combination of two adjacent x_{tk}'s with zeros given to the rest of x_{tk}'s. However, this is not guaranteed by constraints (31) and (33) alone. To obtain a desired result, it is necessary to enforce always only two adjacent variables to stay in the basis of the symplex computation of linear programming.

Reservoir-capacity variable Y_t like plant-capacity variable X_t is replaced by a set of variables $y_{tk}(k = 1, \ldots, n)$ associated with reference capacity K_{sk}.

(37) $Y_t = \Sigma_{k=0}^{n} K_{sk} y_{tk}$ $\qquad t = 1, \ldots, T$

where

(38) $\Sigma_{k=0}^{n} y_{tk} = 1$

$y_{tk} \geq 0$

Using reference capacities K_{sk}, the capital cost $E_s(Y_t)$ and the fixed operating cost $F_{sf}(Y_t)$ of the reservoir are approximated by

$$(39) \qquad \hat{E}_s(Y_t) = \Sigma_{k=0}^{n} e_{sk}\, y_{tk} \qquad t = 1, \ldots, T$$

$$(40) \qquad \hat{F}_s(Y_t) = \Sigma_{k=0}^{n} f_{sk}\, y_{tk}$$

where

$$e_{sk} = E_s(K_{sk}) \qquad k = 0, 1, \ldots, n$$

$$f_{sk} = F_s(K_{sk})$$

Thus, the nonlinear cost functions in X_t and Y_t have been transformed to linear cost functions in x_{tk} and y_{tk}. It is now possible to present a linear program approximating the original problem. The objective function representing the total cost in (29) is rewritten to the following function with x_{tk} and y_{tk} replacing X_t and Y_t:

$$(41) \quad C_{tot} = \Sigma_{t=1}^{T} \frac{1}{R(1+R)^{t-2}} \left(a\, \Sigma_{k=0}^{m} e_{pk}\, x_{tk} + b\, \Sigma_{k=0}^{n} e_{sk}\, y_{tk}\right)$$
$$+ \frac{1}{R(1+R)^{t-3/2}} \left(\Sigma_{k=0}^{m} f_{pk}\, x_{tk} + \Sigma_{k=0}^{m} g_{pk}\, x_{tk} + \Sigma_{k=0}^{n} f_{sk}\, y_{tk}\right)$$
$$- \frac{c\, \Sigma_{i=1}^{t} \Sigma_{k=0}^{m} K_{pk} x_{ik}}{(1+R)^{t-1/2}} - \frac{c\, \Sigma_{i=1}^{T} \Sigma_{k=0}^{m} K_{pk} x_{ik}}{R(1+R)^{T-1/2}} + D \qquad \text{in \$1000}$$

where D is given in (30), and the above C_{tot} is subject to

$$(42) \qquad \Sigma_{k=0}^{m} x_{tk} = 1 \qquad t = 1, \ldots, T$$

$$x_{tk} \geq 0 \qquad k = 0, \ldots, m$$

(43) $\Sigma_{k=0}^{n} y_{tk} = 1$ $\quad t = 1, \ldots, T$

$y_{tk} \geq 0$ $\quad k = 0, \ldots, n$

Similarly, the constraints (21)-(24) are rewritten to the following (44) - (47) in x_{tk}'s and y_{tk}'s:

(44) $\phi\Sigma_{j=1}^{t} \Sigma_{k=0}^{m} K_{pk} x_{jk} \geq Q_t - \phi X_o \quad t = 1, \ldots, T$

(45) $\Sigma_{j=1}^{t}(\phi\alpha\Sigma_{k=0}^{m} K_{pk} x_{jk} + \Sigma_{k=0}^{n} K_{sk} y_{jk})$

$$\geq Q_t^* + \beta Q_t - \phi\alpha X_o - Y_o \quad t = 1, \ldots, T$$

(46) $\frac{10}{24} \phi \Sigma_{j=1}^{t} \Sigma_{k=0}^{m} K_{pk} x_{jk} + \Sigma_{j=1}^{t} \Sigma_{k=0}^{n} K_{sk} y_{jk}$

$$\geq (\gamma-\alpha)\overline{Q}_t + Q_t^* - \frac{10}{24} (\phi X_o - \overline{Q}_t) - Y_o$$

(47) $\Sigma_{j=1}^{t}\Sigma_{k=0}^{n}K_{sk}Y_{jk} \geq (\gamma-\alpha) \overline{Q}_t - Y_o$

Thus, the transformation of the original nonlinear problem to a linear program has been completed. The objective of this linear program is to minimize the total cost C_{tot} in (41) under the set of constraints (42)-(47). Using optimal values of x_{tk} and y_{tk} obtained for the above linear program, values of original variables X_t and Y_t are obtained through transformations (31) and (37). These X_t and Y_t in turn give an approximation to an optimal solution of the original problem. This linear program contains 2T independent variables, T variables of X_t and T variables of Y_t, each being transformed to a unique set of secondary variables representing a polygon. A solution to a linear program such as this usually represents a suboptimum.

Table 3. Estimated Values of Factors Affecting Residential Water Demand in Champaign-Urbana Area for Period 1970-1985

Year	Population[1]	Average Mkt.Value of Dwelling Unit[2]	No.of People Per Dwelling Unit[3]	No. of Dwelling Units	Residential Area in Acres[4]	Dwelling Units Per Acre
t	P	V	p	$a = \frac{P}{p}$	L	$W = \frac{a}{L}$
1970	91,920	13.1	2.545	36,118	5,827	6.198
1971	93,160	13.7	2.554	36,476	6,096	5.983
1972	94,520	14.4	2.563	36,879	6,365	5.794
1973	96,000	15.1	2.572	37,325	6,634	5.626
1974	97,600	15.8	2.581	37,815	6,902	5.479
1975	99,380	16.6	2.590	38,370	7,171	5.351
1976	101,310	17.5	2.599	38,980	7,440	5.239
1977	103,430	18.4	2.608	39,659	7,709	5.145
1978	105,770	19.3	2.617	40,417	7,978	5.066
1979	108,330	20.2	2.626	41,253	8,247	5.002
1980	111,110	21.2	2.635	42,167	8,516	4.952
1981	114,190	22.3	2.644	43,188	8,785	4.916
1982	117,560	23.4	2.653	44,312	9,054	4.894
1983	121,300	24.6	2.662	45,567	9,323	4.888
1984	125,410	25.8	2.671	46,952	9,592	4.895
1985	129,920	27.1	2.680	48,478	9,861	4.916

(1) Data obtained from Champaign-Urbana Area Transportation Study, Interim Report C, January 1968.

(2) Statistical Abstract Supplement, 1967, Department of Commerce, reported $13,700 to be the median value of owner occupied houses comprising 50.2% of all occupied units in the Champaign-Urbana Area in 1967. This value is given a weight of .56 and the 60% of this value, an estimated average value of rented houses, is given .44. The column shows the sum of these weighted values. These values are based on the 1960 census values and used here without adjustments to values in 1964 merely for an illustrative purpose.

(3) Date for 1965 and 1980 obtained from the source described in (1) are interpolated to obtain interim points as indicated.

(4) The transportation study referred to in (1) gives 4,482.3 acres for 1965 and 9,861.4 acres for 1985. These values are linearly interpolated to find interim points.

Chapter 4

APPLICATION OF CAPACITY EXPANSION MODEL TO CHAMPAIGN-URBANA AREA

To illustrate how the linear programming model developed in the previous section can be applied to a specific locality, the Champaign-Urbana twin-city area is selected.

It is assumed that, as of the beginning of 1970 the existing water treatment-distribution system serving the twin city community is composed of 2 treatment plants, each with a capacity of 10 mgd, and 5 distribution reservoirs, each with a capacity of 1 mg.[1] This system needs a long-range plan for its capacity expansion to cope with continuously growing demand over the next 16 years after which demand is expected to stay constant at the level in the last year of the 16-year period. Water is obtained from underground sources which can accommodate the requirements of the area for an indefinite future.

First, we need estimate future demands for residential water in the area. The demands have been forecast through Eqs. (1)-(3) using estimated values of factors such as population, average market value of a dwelling unit, number of people in dwelling unit, number of dwelling units, and residential area in the region. The estimated values of the factors and their sources are listed in Table 3. For example, the estimates of population in various years are obtained from Champaign-Urbana Area Transportation Study covering the 16-year period between 1970 and 1985. This period will be adopted to the present study as the planning period.

The average market values of dwelling units are estimated from data in Statistical Abstract Supplement - 1967, published by the U. S. Depart-

[1]These capacities approximate the existing system supplying water to the twin-city area.

Table 4. Estimated Water Requirements in Champaign-Urbana Area for Period 1970-1985

Year t	Expected Average Demand (1) mgd $\overline{Q}_t$	1.5 times $\overline{Q}_t$ mgd $1.5\ \overline{Q}_t$	Expected Maximum Demand (2) mgd $\overline{Q}_{(mxdy)t}$	Maximum Daily Requirement(3) mgd Q_t	Fire Fighting Requirement (4) mg Q_t^*
1970	14.42	21.63	20.59	21.63	5.30
1971	14.97	22.46	21.48	22.46	5.33
1972	15.54	23.31	22.40	23.31	5.37
1973	16.12	24.18	23.32	24.18	5.40
1974	16.71	25.07	24.25	25.07	5.44
1975	17.33	26.00	25.21	26.00	5.49
1976	17.95	26.93	26.18	26.93	5.53
1977	18.63	27.95	27.20	27.95	5.59
1978	19.31	28.97	28.21	28.97	5.64
1979	20.01	30.02	29.24	30.02	5.70
1980	20.74	31.11	30.30	31.11	5.77
1981	21.51	32.27	31.39	32.27	5.84
1982	22.30	33.45	32.50	33.45	5.91
1983	23.14	34.71	33.64	34.71	5.99
1984	24.02	36.03	34.82	36.03	6.08
1985	24.95	37.43	36.04	37.43	6.18

(1) The value is determined by Eq. (1) with data in Table 3.

(2) The value is determined by Eq. (2) with data in Table 3.

(3) The value represents a larger one of $1.5\ \overline{Q}_t$ or $\overline{Q}_{(mxdy)t}$.

(4) The value is determined by Eq. (8) with data in Table 3.

ment of Commerce. According to this report, owner occupied houses comprise 50.2% of all occupied units in the twin-city area and their median value was \$13,700 in that year. It is assumed that this median value also represents the mean value of the owner occupied units, and that 60% of this value, or \$8,220, approximates the average of the rented units. The sum of these values weighted by .56 and .44, the proportions of the owner occupied and rented units to all dwelling units, is \$11,300 and this value represents an approximate average of the values of all dwelling units. With this value in 1967 as the starting point, the market values in each year is obtained by increasing the previous year's value at an assumed rate of 5%. Finally, the number of people per dwelling unit is obtained by a linear interporation of 2.5 in 1965 and 2.68 in 1985 reported by the previously quoted Champaign-Urbana Area Transportation Study.

Those estimated values as listed in Table 3 are used to determine various water requirements during the 16-year period shown in Table 4. In this table, the expected average demand in each year is determined by Eq. (1) and the expected maximum demand by Eq. (2). Then following Eq.(19) suggested by the American Insurance Association, a larger one of 1.5 times the expected average demand or the expected maximum demand is selected as the estimated maximum daily requirement, Q_t. Finally, the fire-fighting requirement, Q_t^*, is determined by Eq. (8).

Linaweaver [22, p. 100] illustrated a diagram showing demand fluctuations in a typical summer week and those in a typical winter week in Creekside Acres, Oakland, California; a part of this diagram has been presented in Figure 1. From the diagram, we have estimated the following α, β, and γ for the length of the peak period on a typical day, the demand of the peak period on the maximum demand day, and this demand on the average demand day as fractions of the total length and demand of the respective days:

$$\alpha = .6, \ \beta = .91, \ \gamma = .85$$

We assume these values also represent the respective relations in the twin-city area.

Various costs included in the objective function of the linear program, Eq. (41), are determined by the cost functions in (9) and (13)-(18) which are summarily listed as follows:

A. Cost functions of the ground water treatment plant

(1) Capital Cost $\quad E_p(K_p) = 115\ K_p^{.63}$ in \$1000

(2) Fixed operating cost $\quad F_p(K_p) = \delta(34.79\ K_p^{.63})$ in \$1000/yr

(3) Variable operating cost at capacity operation $\quad G_p(K_p) = \delta(34.79\ K_p^{.63} - 5.06\ K_p^{1.02})$ in \$1000/yr

where K_p is the capacity of the plant in mgd and $\delta = .43$.

B. Cost functions of the ground level distribution reservoir

(1) Capital cost $\quad E_s(K_s) = 128\ K_s^{.75}$ in \$1000

(2) Fixed operating cost $\quad F_s(K_s) = .75\ K_s^{.71}$ in \$1000/yr

where K_s is the capacity of the reservoir in mg.

C. Cost of pumping

$$C_{pump} = 31.4\ c/E_o \quad \text{per 1000 gallons/100 ft}$$

where c is \$ per kw-hr and E_o is the wire-water efficiency in percent.

With regard to the distribution reservoir, the objective function in (41) includes only the capital and fixed operating costs; the variable operating cost is assumed independent of the size of the reservoir but dependent on the volume of water required for equilization on an average demand day. With this assumption, d in the constant term D of the objective function (41) is replaced by the following specific unit cost determined by the above cost of pumping:

$$d = 6.28 \qquad \$/\text{mg}$$

where we have assumed the rate of electricity is 1 cent per Kw-hr, the pumping height is 100 ft., and the wire-water efficiency is 50%.

The range of capacity of the treatment plant being considered is from 0 mgd to 30 mgd, and this range is divided to 15 equal segments and represented by 16 reference points. Likewise, the range of capacity of the ground reservoir being considered is from 0 mg to 3.0 mg and this range is represented by 16 reference points separated by an identical step of .20 mg. Table 5 lists the capital and operating costs of the treatment plant and ground-level reservoir at the above capacity reference points and the capacity variables representing these reference points.

The linear program incorporating the above conditions has been computed by MPS, an IBM linear programming code, on an IBM 360/75, using each of the following values of the booster pumping coefficient: ϕ = 1.0, 1.1, 1.2, 1.3, and 1.4. The results of the computations are listed in Table 6, showing for each value of ϕ the times at which new facilities are installed, reference capacities and their variables

Table 5. Capital and Operating Costs of Ground-Water Treatment Plant and Ground-Level Distribution Reservoir at Various Capacity-Reference Points.

Costs of Ground-Water Treatment Plant						Costs of Ground-Level Distribution Reservoir				
Reference Point	Capacity Variable	Capacity (mgd)	Capital Cost ($1000)	Fixed Operating Cost ($1000/yr)	Variable Operating cost at u=1	Reference Point	Capacity Variable	Capacity (mg)	Capital Cost ($1000)	Fixed Operating Cost ($1000/yr)
k	x_k	K_{pk}	$E_p(K_{pk})$	$F_p(K_{pk})$	$G_p(K_{pk})$	k	y_k	K_{sk}	$E_s(K_{sk})$	$F_s(K_{sk})$
0	x_0	0	0.000	0.000	0.000	0	y_0	0.000	0.000	0.000
1	x_1	2	177.970	23.151	18.739	1	y_1	0.200	38.281	0.239
2	x_2	4	275.420	35.828	26.880	2	y_2	0.400	64.381	0.391
3	x_3	6	355.576	46.255	32.724	3	y_3	0.600	87.262	0.522
4	x_4	8	426.230	55.446	37.300	4	y_4	0.800	108.275	0.640
5	x_5	10	490.566	63.815	41.032	5	y_5	1.000	128.000	0.750
6	x_6	12	550.278	71.583	44.143	6	y_6	1.200	146.756	0.854
7	x_7	14	606.399	78.883	46.771	7	y_7	1.400	164.743	0.952
8	x_8	16	659.619	85.806	49.008	8	y_8	1.600	182.096	1.047
9	x_9	18	710.427	92.415	50.920	9	y_9	1.800	198.913	1.138
10	x_{10}	20	759.183	98.758	52.555	10	y_{10}	2.000	215.269	1.227
11	x_{11}	22	806.165	104.869	53.949	11	y_{11}	2.200	231.221	1.313
12	x_{12}	24	851.591	110.779	55.132	12	y_{12}	2.400	246.813	1.396
13	x_{13}	26	895.635	116.508	56.128	13	y_{13}	2.600	262.083	1.478
14	x_{14}	28	938.442	122.077	56.956	14	y_{14}	2.800	277.063	1.558
15	x_{15}	30	980.131	127.500	57.631	15	y_{15}	3.000	291.777	1.636

Table 6. Application of Capacity Expansion Model to Champaign-Urbana Area: Determination of Optimum Capacities of Treatment Plants and Distribution Reservoirs Under Various Degrees of Booster Pumping.

Case (Coefficient of Booster Pumping)	Types of Facilities	Order of Install.	Year of Install. t	Reference Capacities (Lower) K_{pL} or K_{sL}	Reference Capacities (Upper) K_{pU} or K_{sU}	Optimum Values of Variables (Lower) x_{tL} or y_{tL}	Optimum Values of Variables (Upper) x_{tU} or y_{tU}	Capacity $K_{pL}x_{tL}+K_{pU}x_{tU}$ or $K_{sL}y_{tL}+K_{sU}y_{tU}$	Total Cost of Expansion Plan* ($1000)
Case 1 (ϕ = 1.0)	Treatment Plants	1	1970	24 mgd	26 mgd	.5016	.4984	24.997 mgd	1826 +D
		2	1979	10	12	.1658	.8342	11.669	
	Distribution Reservoirs	1	1979	.4 mg	.6 mg	.1250	.8750	.571 mg	
		2	1983	.6	.8	.6875	.3125	.663	
Case 2 (ϕ = 1.1)	Treatment Plants	1	1970	18 mgd	20 mgd	.2880	.7120	19.424 mgd	1715 +D
		2	1978	8	10	.6279	.3721	8.744	
	Distribution Reservoirs	1	1979	.6 mg	.8 mg	.0750	.9250	.785 mg	
		2	1984	2.6	2.8	.6970	.3030	2.661	
Case 3 (ϕ = 1.2)	Treatment Plants	1	1970	16 mgd	18 mgd	.9306	.0694	16.139 mgd	1597 +D
		2	1978	8	10	.9922	.0078	8.016	
	Distribution Reservoirs	1	1979	.6 mg	.8 mg	.0750	.9250	.785 mg	
		2	1984	2.6	2.8	.6970	.3030	2.661	
Case 4 (ϕ = 1.3)	Treatment Plants	1	1970	12 mgd	14 mgd	.3206	.6794	13.359 mgd	1483 +D
		2	1978	6	8	.3005	.6995	7.399	
	Distribution Reservoirs	1	1979	.6 mg	.8 mg	.0750	.9250	.785 mg	
		2	1984	2.6	2.8	.6970	.3030	2.661	
Case 5 (ϕ = 1.4)	Treatment Plants	1	1970	10 mgd	12 mgd	.5120	.4880	10.976 mgd	1374 +D
		2	1978	6	8	.5648	.4352	6.870	
	Distribution Reservoirs	1	1979	.6 mg	.8 mg	.0750	.9250	.785 mg	
		2	1984	2.0	2.8	.6970	.3030	2.661	

*This total cost represents the sum of capital and operating costs associated with facilities installed, discounted at an annual interest rate of 10%. All component costs are at the 1964 price level. D listed in this column represents the constant cost defined by (30) in p. 27 and equals $1,353,000.

representing the facilities and the total cost.

For each value of ϕ, several computer runs have been made by fixing the number of plants being installed to 2 and the number of reservoirs to 2. These numbers have been set at 2, because 1 plant or 1 reservoir requires a capacity exceeding the predetermined range and test runs with 3 plants or 3 reservoirs have produced less desirable results than those for 2 plants or 2 reservoirs.

The solutions listed in Table 6 represent those giving the minimum costs among the results obtained. Since a solution to this linear program usually is a suboptimum, those solutions listed in Table 6 probably represent suboptima. Naturally, the required capactiy of each plant decreases as the value of ϕ increases; the decrease in capacity is more pronounced with the first plant than with the second plant. On the other hand, the value ϕ does not affect the capacities of the selected reservoirs which are identical in all cases except for case 1.

The total cost in Table 6 represents the sum of the present values of capital and operating cost associated with new facilities, discounted at an annual rate of 10%. As is expected, the total cost decreases with an increasing ϕ, since the capacity of a plant expands with an increasing ϕ without an additional expense. The total costs in Table 6 alone can not determine the relative merits of the five cases, since they don't include such items as booster pumps, maintenance and repairs, and operating problems related to variable degrees of booster pumping.

Table 7. Application of Capacity Expansion Model to Champaign-Urbana Area: Capacities of Treatment Plants and Distribution Reservoirs.

Case	Booster Pumping ϕ	Plant Capacity and Year of Installation 1st mgd(yr.)	Plant Capacity and Year of Installation 2nd mgd(yr.)	Reservoir Capacity and Year of Installation 1st mg(yr.)	Reservoir Capacity and Year of Installation 2nd mg(yr.)
Case 1	1.0	25.0(1970)	11.7(1979)	.57(1979)	.66(1983)
Case 2	1.1	19.4(1970)	8.7(1978)	.78(1979)	2.66(1984)
Case 3	1.2	16.1(1970)	8.0(1978)	.78(1979)	2.66(1984)
Case 4	1.3	13.4(1970)	7.4(1978)	.78(1979)	2.66(1984)
Case 5	1.4	11.0(1970)	6.9(1978)	.78(1979)	2.66(1984)

Finally, the purpose of this application is to show how the linear program developed can be applied to a specific locality rather than to suggest a plan to be used for the selected region. The latter is particularly true in view of the crude cost functions used for this application. However, where reliable current information is available for both forecasting future demands and determining the capital and operating cost functions for the specific types of facilities considered for installation, the linear programming model will be useful in drafting an initial plan for the capacity expansion of a municipal water system.

CHAPTER 5

SUMMARY

A rapid increase in population accompanied by sustained improvement in living standards has stimulated demands for water in residential use in most cities. To assure adequate supply of water for the existing and future demands, a municipal water treatment-distribution system might be required to add new treatment plants and/or distribution reservoirs to the existing water treatment-distribution system. Since these facilities are expensive to build and operate, the capacity expansion calls for a careful plan based on sound analysis that takes into consideration all relevant engineering and economic factors.

Like most capital facilities, the treatment plant or the distribution reservoir is subject to economies of scale. Namely, the bigger the capacity of a facility, the smaller the capital or operating cost per unit volume of water treated or stored. This statement is normally valid within a certain practical range of capacity. Where demands are constantly increasing, the above scale effects dictate us to install a facility that satisfies not just the immediate needs but also the requirements beyond the immediate future.

The design capacity of a municipal water supply system depends on two factors: one of them is the expected maximum daily demand influenced mainly by lawn sprinkling and airconditioning on hot summer days, and the other the fire-fighting requirements recommended by the American Insurance Association. With regard to the rate of demand, an average day is divided to two periods, the peak period starting around 7 a.m. and ending around 9 p.m. and the slack period covering the rest of the day. However, the treatment plant is usually operated at a constant rate throughout the day so as to eliminate costly changes in output rate and to minimize the required plant capacity. In this mode of operation, the plant pumps out surplus water during the slack period that is accumulated in distribution storage and will be used for compensating the short supply during the subsequent peak period.

The formulation of the capacity expansion model assumes several conditions. The scale effects in the capital and operating costs of a treatment plant or a distribution reservoir are represented by concave functions of capacity in exponential form. Demand, that can be forecast with certainty, continuously increases over a finite period beyond which it stays at the maximum level attained at the end of the period. This period is adopted as the planning period for installing new facilities. The facilities existing at the outset and those installed during the planning period are replaced by permanent chains of facilities identical with them.

The model formulated is composed of an objective function minimizing the total cost of investment and operation and a set of constraints on the treatment and storage capacities satisfying the annual requirements on the expected maximum daily demand, equilization during the daily peak period, and fire-fighting. The original non-linear formulation is transformed to a linear program by replacing each concave cost function in the objective function with a set of linear functions approximating the cost between successive reference capacity points.

The design capacities determined by the formulas suggested by various authors tend to be much greater than the capacity used in practice. This discrepancy is resolved by multiplying every capacity variable in the formulation with a coefficient of booster pumping having a value equal to or larger than unity. The determination of a specific value rests on the discretion of an individual user of the model.

The linear programming model thus developed has been applied to the Champaign-Urbana area, Illinois, to illustrate how it can be used in practice. Because many conditions used have to be assumed, this application is for illustrating the steps involved in using the model rather than for suggesting a plan to be adopted. The period 1970-1985 has been used for forecasting demands and planning the installation of new facilities. Solutions to the linear program have been computed with five different values given to the coefficient of booster pumping. The solutions have invariably selected two treatment plants and two distribution reservoirs, indicating that an increase in the value of the coefficient accompanies a greater decrease in the capacity of the first plant but a smaller decrease in that of the second plant. Further, the facilities are installed in the same years despite the different values of the coefficient.

Finally, the usefulness of the model developed in this study depends much on the availability of reliable information on future demands and the capital and operating costs of the types of facilities considered. Since most cost data available in published studies are not in the form useful to the model, the collection of reliable current data for determining accurate cost functions must precede the actual use of this model. Most capital investment decisions are based on trade-offs between the cost of over capacity and the penalty of under capacity for given requirements. In the present problem, the former is identifiable, but the latter, the penalty of under capacity, is a very vaguely structured concept. For this penalty would come to the consumer of water, rather than to the supplier, in the forms of higher fire insurance rates, shortage in supplied water, or poorly treated water. If and when we could identify the relationship between the under capacity and its penalty imposed on the water supplying agency, the model formulated here would become a better and more useful tool for the water supplying agency in making a capacity expansion decision.

APPENDIX I

ON COSTS OF SURFACE-WATER TREATMENT

Introduction

Published studies on water treatment costs are extremely scarce and, when available, they usually show the average total costs per gallon of water treated by plants with specific capacities operated at particular utilization rates. Such average costs are useful for comparing relative operating efficiencies of different plants or determining profit margins per gallon of water sold under the given conditions. If the utilization rates change, so do the average costs. Therefore, the average costs determined for specific utilization rates supply limited information on the costs incurred by the same plants operating at different utilization rates. In such a case, one needs a cost formula expressed as a function of utilization rate. Further, if he were to select an optimum capacity for a plant that would operate at various utilization rates, it would be essential for him to have a cost formula described as a function of plant capacity and utilization rate.

Proposed Cost Function

Like most capital facilities, water-treatment plants are subject to economies of scale related to their sizes or capacities. Theoretically, the relationship between capacity and captial cost or capacity and total cost of operation at the rated capacity may be described by an elongated inverse-S shape, a concave-convex function, as is illustrated in Figure 2. In the concave region shown by curve AB, the capital cost or the total operating cost increases with an increase in capacity but at a decreasing rate; in the convex region shown by curve BC, however, those costs increase at an increasing rate as the capacity increases. From the economic point of view, the investor should be interested in the concave region but not in the convex region unless an increase in the cost due to an added capacity is completely counterbalanced by decreases in other costs. Economies of scale represented by the concave region are the main underlying reason in many cases why a larger facility is preferred to a set of smaller ones to do the same task or why a larger facility is installed instead of a smaller one satisfying immediate requirements when output requirements are expected to increase with time. A reasonable form of cost function giving the above scale effects is the following exponential function proposed by Chenery.[1]

(i) $$Y = aK^b$$

where Y is the cost, K is rated capacity and a and b are parameters associated with a particular type of facility. This exponential form will later be applied to the capital cost and the total operating cost.

The total annual cost of water treatment proposed here is composed of capital cost and operating cost, and is written as a function of plant capacity and utilization rate as follows:

(ii) $$C(K,u) = \alpha E(k) + F_t(K,u) \qquad (\$1000/mg)$$

where

C(K,u): the total annual cost of water treatment for a plant with capacity K operated at utilization rate u ($1000/mg).

K: the plant capacity for a 24-hour operation (mgd).

u: the utilization rate, or the volume of water processed per day expressed as a fraction of the rated plant capacity.

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1. Chenery, H.B., "Overcapacity and the Accleration Principle," ECONOMETRICA, 20: 1-28, January, 1952.

α: the amortization factor determined by the interest rate and the number of years to write off the original investment. For example, α = .05783 if the plant is amortized over 30 years at a rate of 4%.

$E(K)$: The capital cost of a plant with capacity K ($1000).

$F_t(K,u)$: The total annual operating cost for a plant with capacity K operated at utilization rate u ($1000/yr).

Specific forms of $E(K)$ and $F_t(K,u)$ on the right hand of (2) will be determined using data available in Koenig's study [21] and publication of the Illinois State Water Survey (ISWS) [16] Among the publications examined by the author, those were the only sources that provided useful data for the present study.

Capital Cost

The capital cost required for the installation of a plant contributes the greatest part to the annual treatment cost. When the capital cost is amortized over a period of 30 years at 4% interest rate, the contribution is estimated as 40-55% of the treatment cost depending on the rate of plant-capacity utilization [16,p. 324]. Further, sources from which water is obtained greatly influence the capital cost of water treatment. The following discussion is based on an ISWS report on surface-water treatment cost [16, Tech. Letter 11].

The total treatment cost of surface water reported by Koenig is based on data from 30 plants. The capital cost of a plant covers the low lift pumping station, the treatment plant itself, and the high lift pumping station, but it does not include conveyance lines for raw water or finished water, nor booster stations on finished water lines

[21, p. 295]. ISWS adjusted data from 42 plants (including Koenig's 30 plants and other data which appeared in JAWWA to 1964 prices and to location differences by using the Handy-Whitman Utilities Indexes for small treatment plants [0 to 1 million gallons per day (mgd)] and large treatment plants (greater than 1 mgd). Using the adjusted data, ISWS then obtained the following regression relationship between capacity and capital cost:

$$\text{(iii)} \qquad E(K) = 267.9K^{0.65} \qquad (\$1000)$$

where E(K) is the capital cost of a surface-water treatment plant and K is the capacity in mgd. The exponent b = .65 in (iii) means that the capital cost increases with an increase in capacity at a positive but decreasing rate, therefore E(K) in (iii) will have a concave curve such as curve AB in Figure 1.

Operating Cost

Like the capital cost, the operating cost of a water treatment plant is affected by the plant capacity, utilization rate, and sources of water. The last factor is not given special consideration in the present discussion, since Koenig's study merely indicates that the plants surveyed treat surface water. The study reports that the largest component of the annual treatment cost is the capital cost contributing 40-55% in typical plants; the next major item is manpower, contributing 22% in typical plants; and the third item in the list is energy with 10-13% contributions. These three items contribute almost 7/8 of the total

Table I. Elements of Average Water Treatment Cost in "Typical Plants" (1964 Price Level)

Design Capacity K mgd	0.5		8.0	
Utilization Rates u	0.5	1.0	0.5	1.0
Item	$/mg	$/mg	$/mg	$/mg
Manpower	67.0	61.0	27.0	16.0
Maintenance, Repair and Replacement	7.0	5.0	2.9	1.7
Miscellaneous	2.2	1.1	2.2	1.1
Heating (140 days)	9.2	4.6	2.2	1.1
Energy	33.0	33.0	17.0	17.0
Chemicals	18.5	18.5	7.2	7.2
Average Operating Cost f(u)	136.9	123.2	58.5	44.1

Source: Koenig /_ 3, p. 324_/

treatment cost. Other items included in the cost are chemicals with 6% contribution, heating, maintenance, and repair, each with 2% contribution. The total operating costs of "typical plants" tabulated by Koenig [p.324] is rearranged and reproduced in Table I, in which the

the heating cost represents about the maximum to be experienced in the United States.

Using data in Table I and assuming that the variable cost is a linear function of utilization rate, the fixed and variable elements of the total operating cost will be identified.

First, two individual values listed under each design capacity in Table 1 are combined to a single expression with or without utilization rate u, as listed in Table II. To obtain the original values in Table I, we need merely to replace u in the expressions in Table II with .5 or 1.

Table II. Classification of Elements of Average Water Treatment Cost in "Typical Plants"

Design Capacity K mgd	0.5	8.0	
Items	\$/mg	\$/mg	Classification
Manpower	$\frac{6.0}{u} + 55.0$	$\frac{11.0}{u} + 5.0$	Semi-variable
Maintenance, Repair and Replacement	$\frac{2.0}{u} + 3.0$	$\frac{1.2}{u} + 0.5$	Semi-variable
Miscellaneous	$\frac{1.1}{u}$	$\frac{1.1}{u}$	Fixed
Heating (140 days)	$\frac{4.6}{u}$	$\frac{1.1}{u}$	Fixed
Energy	33.0	17.0	Variable
Chemical	18.5	7.2	Variable
Average Operating Cost f(u)	$\frac{13.7}{u} + 109.5$	$\frac{14.4}{u} + 29.7$	

Note: u represents the plant utilization rate.

In Table II, an item is classified as a variable, fixed, or semi-variable element of the operating cost depending on whether it is given by a definite value only, a fractional form with u as a denominator, or a combination of a definite value and a fraction with u. Therefore, "Manpower" and "Maintenance and Others" are semi-variable elements, "Miscellaneous" and "Heating" are fixed elements, and "Energy" and "Chemical" are variable elements.

The average operating cost at the bottom of Table II is given by a fraction, with u as the denominator, representing the fixed part of the operating cost and a constant value representing the variable part. With those average operating costs for K = .5 and 8, we wish to determine general formulas for the fixed and variable operating costs as functions of capacity.

First, we obtain the annual fixed and variable costs for those capacities by multiplying with 365 uK (K = .5 or 8) those parts of the average cost in Table II. The results of multiplications give the following fixed operating costs F_f (K) for K = .5 and 8 mgd.

$$F_f(K = .5) = 365uK\frac{13.7}{u} = 2{,}500 \qquad (\$/Yr)$$

$$F_f(K = 8) = 365uK\frac{14.4}{u} = 42{,}048 \qquad (\$/Yr)$$

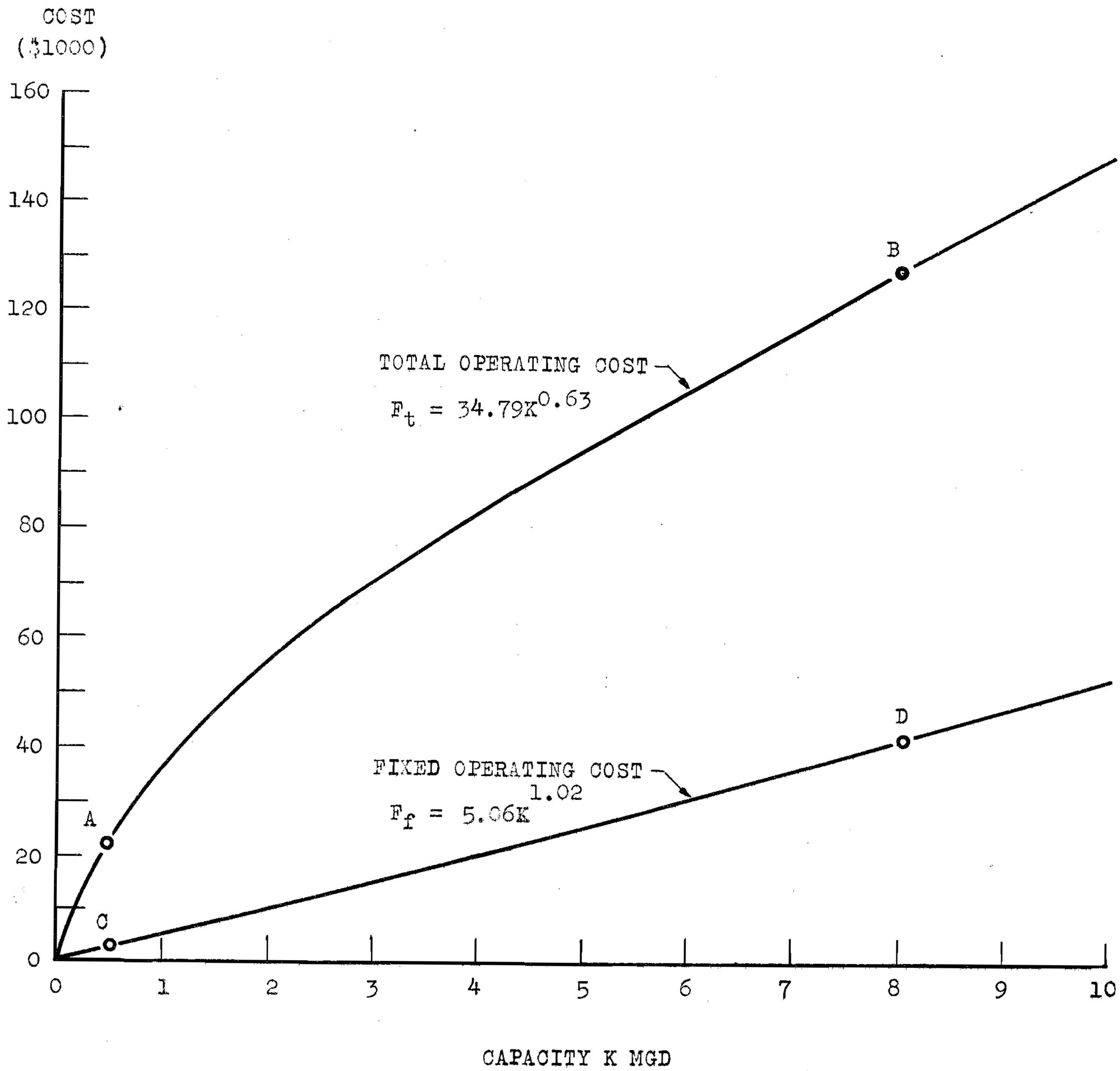
The variable operating costs for K = .5 and 8 mgd are

$$F_v(K = .5,u) = 365uK(109.5) = 19{,}984u \quad (\$/Yr)$$

$$F_v(K = 8,u) = 365uK(29.7) = 86{,}724u \quad (\$/Yr)$$

The annual total operating costs for capacity operation are given by the sums of the above fixed costs and variable costs (with u = 1) for the respective design capacities:

FIGURE I. ESTIMATION OF FUNCTIONS FOR TOTAL AND FIXED OPERATING COSTS OF SURFACE-WATER TREATMENT BASED ON KOENIG'S DATA



$F_t(K = .5,\ u = 1) = 2{,}500 + 19{,}984 = 22{,}484$ (\$/Yr)

$F_t(K = 8,\ u = 1) = 42{,}048 + 86{,}724 = 128{,}772$ (\$/Yr)

In Figure I , points A and B represent the total operating costs, \$22,484 and \$128,772 and points C and D the fixed operating costs, \$2,500 and \$42,048. Functions whose curves pass through the above pairs of points determine the relationships between capacity and the annual total and fixed costs of operation. To reflect scale effects, these functions are formulated to exponential functions of the form shown in (i); specifically they are given by the following (iv) and (v) representing the total operating cost for capacity operation and the fixed operating cost, respectively:

(iv) $F_t(K,\ u = 1) = 34.79K^{0.63}$ (\$1000/Yr)

(v) $F_f(K) = 5.06K^{1.02}$ (\$1000/Yr)

The exponent of K in (iv) equals 0.63 showing an increase in capacity accompanies a positive but less-than proportional increase in the total operating cost for capacity operation. The exponent of K in (v) is almost equal to 1, meaning the fixed operating cost is approximately a linear function of capacity. Thus, according to Koenig's data, economies of scale in water treatment are realized by savings in variable operating cost and capital cost, but not in fixed operating cost.

Since the variable operating cost is assumed as a linear function of utilization rate u, it is given by the product of u and the difference between the total cost and the fixed cost:

(vi) $F_V(K,u) = u(34.79K^{0.63} - 5.06K^{1.02})$ ($1000/Yr)

Using **(v)** and **(vi)**, the total operating cost is obtained as a function of K and u:

(vii) $F_t(K,u) = 5.06\,K^{1.02}\,(1 - u) + 34.79K^{0.63}u$ ($1000/Yr)

Using **(vii), it is now possible to determine the total operating** cost of water treatment for a given K and u. For example, setting K = 4 mgd and u = 0 or 1 **in (vii), we obtain the total operating** costs for respective utilization rates as follows:

$F_t(K = 4,\ u = 0) = 5.06(4^{1.02}) = 20.764$ ($1000/Yr)

$F_t(K = 4,\ u = 1) = 34.79(4^{0.63}) = 83.244$ ($1000/Yr)

Similarly, the total operating costs for K=1, 2, ..., 10 mgd are computed and shown graphically in Figure 3, where points E and F represent the costs computed above for K=4 mgd and u=0 and 1. As the utilization rate increases from 0 toward 1, the total operating cost increases from E representing the fixed operating cost to F representing the total cost of operation at capacity, along the straight line EF because of the assumption that the variable cost is a linear function of utilization rate. For example, the total cost at u=0.7, shown by point G, is obtained by using **(vii) as follows:**

$F_t(K=4,\ u=0.7) = 5.06\,(4^{1.02})\,(1-0.7) + 34.79(4^{0.63})0.7 = 64.500$

($1000/Yr)

Finally the total cost of water treatment is obtained by substituting E(K) in (iii) and $F_t(K,u)$ in (vii) into (ii):

$$\text{(viii)} \quad C(K,u) = \alpha(267.9K^{0.65}d) + 5.06\ K^{1.02}\ (1-u) + 34.79K^{0.63}\ u \quad (\$1000/Yr)$$

where α is an amortization factor to convert the capital cost into equivalent annual payments over the life of the treatment plant.

Conclusion

Information regarding costs of water treatment is extremely difficult to find. The average costs of water treatment usually found in published studies may be useful for comparing the relative efficiencies of various operations using the specified plant capacities and utilization rates. If operations involve plant capacities or utilization rates different from the specified values, those average costs fail to give useful information.

Because of economies of scale available in large plants, the capacity of a new plant usually exceeds the immediate requirements. To find an economically optimal capacity for such a plant, often for increasing requirements with time, costs of operation must be estimated for various capacities and utilization rates by using some form of cost function. Such a cost function may serve the purpose of estimating the orders of magnitude of water treatment costs under various conditions, but it does not provide information on specific costs for plants to operate under particular conditions.

The cost function proposed here is based on data appeared in publications of Koenig and the Illinois State Water Survey and may be valid as far as these data are concerned. When other data become available or a plant is to operate under specific conditions, a new function may be formulated using the method suggested in this paper.

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